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19 AUG 86 N00014-84-K-0499

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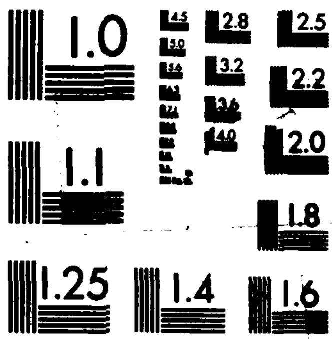

MICROCOPY RESOLUTION TEST CHART
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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Research on both internal and free-surface flows related to experimental situations was continued. Part of this research was based on using a low-order code developed previously, jointly by Pritchard and Scott. Funds for the programmer allowed two substantial new programming projects to be initiated during this time. One concerned investigation of techniques for implementing a quartic finite element method for viscous, incompressible, two-dimensional flows. The other involved a streamline-diffusion finite element method for inviscid flows.

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Figure 8. Report Documentation Page, DD Form 1473 (1 of 2)

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Figure 8. Report Documentation Page, DD Form 1473 (2 of 2)

Submitted 8/19/86 021778

FINAL REPORT

"Computational and Experimental Fluid Dynamics"

ONR Grant for the period June 1984 through September 1985

Funds were used for two major categories: support for released time for the principal investigator and salary for a programmer. The former allowed the principal investigator to visit both Los Alamos National Laboratory and the Mathematics Research Center at the University of Wisconsin during the winter and spring of 1985, as well as providing 50% released time during the fall of 1984. This facilitated the research, especially with regard to collaboration with W. G. Pritchard, who was visiting the Mathematics Research Center during that time. Research on both internal and free-surface flows related to experimental situations was continued. Part of this research was based on using a low-order code developed previously, jointly by Pritchard and Scott. Funds for the programmer allowed two substantial new programming projects to be initiated during this time. One concerned investigation of techniques for implementing a quartic finite element method for viscous, incompressible, two-dimensional flows. The other involved a streamline-diffusion finite element method for inviscid flows.

The collaboration with Pritchard during this time period focused on two experimental situations. One involves internal flow in a complicated geometry (see Figure 1, which includes an indication of a typical mesh used in computations) that models the critical part of a so-called "stress meter" for determining rheological properties of non-Newtonian fluids. Experimental work on this was done by Pritchard and Arthur Lodge of the University of Wisconsin. A pleasant result of this part of the research during this period was that we discovered a manufacturing flaw in a new meter by using the numerical code to predict a particular stress difference. A second result of this research is a series of comparisons of different numerical methods, done jointly by David Malkus and John Strikwerda of the University of Wisconsin together with Pritchard and Scott. It is hoped that this will help to establish the "stress meter" model problem as a benchmark with a firmer physical foundation than the frequently-used "driven cavity" problem.

The typical quantity of interest in the problem depicted in Figure 1 is the normal stress difference between the bottom of the slot, or hole, in the middle of the figure and the top of the channel above the hole. This differnce is zero for Reynolds number equal to zero, and it is plotted as a function of Reynolds number in Figures 1a and 1b for different hole depths (Figure 1 depicts a hole depth of 1.0). Note that the stress difference decreases monotonically, as a function of depth, for depths down to 0.75, however, for a depth of 0.5 it suddenly jumps up. This needs to be studied in more detail.

The other experimental phenomena under study during this period involved free-surface flow. Pritchard had recently completed a series of experiments at the University of Essex, and work was begun to modify the code in order to allow for a free surface. Figure 2 shows a typical plot of the experimental data; note the complexity of the free surface, even though this is a relatively low Reynolds number. Our goal will then be to compare the new codes predictions with the results of the physical experiments and then to study various free-surface phenomena.

The above work is being written up for publication in a series of papers. Tentative titles for these are as follows:

W. G. Pritchard, Y. Renardy and L. R. Scott, "Tests of a numerical method for viscous, incompressible flows. I: Fixed-domain problems," (in preparation).

W. G. Pritchard, Y. Renardy and L. R. Scott, "Tests of a numerical method for viscous, incompressible flows. II: Free-surface problems," (in preparation).

A third publication is planned jointly by Malkus, Pritchard, Scott and Strikwerda, but a title has not yet been chosen.

Although the codes developed by Pritchard and Scott will solve a wide range of problems, it was also desired to develop entirely new codes implementing new numerical techniques. Two independent directions were pursued during this period. One involved higher-order spatial accuracy and better representation of the incompressibility constraint in the numerical discretization based on implementing a quartic finite element method that has been studied theoretically by Scott and Michael Vogelius (of the University of Maryland). The other addressed temporal discretization, especially with regard to high-Reynolds-number flows, based on a streamline-diffusion finite-element technique developed by Claes Johnson of the University of Gothenberg when he was visiting the University of Michigan. To achieve these goals most cost-effectively, it was decided to hire a professional programmer rather than rely solely on Pritchard and Scott (and other collaborators) for the coding, as had been done in the past. This approach is quite standard in governmental and industrial laboratories, but it is somewhat unusual in academic circles. Although this approach was complicated, primarily due to its novelty in a university setting, it was successful and would be recommended for future projects under appropriate circumstances. (The availability of an appropriate person is critical; our programmer left the university at the end of this project to work at an industrial laboratory at a 50% increase in salary and would not be enticed back to the academic setting easily. Moreover, the demand for such people in industry far exceeds the supply.)

The numerical studies with the quartic finite-element method for viscous, incompressible, two-dimensional flows were based on a modification of the IMSL code TWODEPEP. Two approaches were followed. The first, which involved a straightforward use of the

code, tested the standard penalty method for imposing the incompressibility constraint. Typical results of this study are given in Tables 1 and 2. A preliminary description of them was given by Scott in the conference paper "Spatial discretization techniques for the Navier-Stokes equations: theoretical and computational results," which appeared in the proceedings entitled COMPUTING METHODS IN APPLIED SCIENCES AND ENGINEERING VI (R. Glowinski and J.-L. Lions, eds., Amsterdam: North-Holland, 1984, 451-470). Note that the results in Table 1 indicate the high order of accuracy of the method, at least in the L1 norm. (The poblem being solved, as shown in Figures 3a and 3b, is not sufficiently regular to prove optimal-order accuracy, due to the lack of smoothness of the boundary.) The data in Table 1 indicates only 4th order convergence in the L2 norm, and not the best-possible 5th order convergence that approximation theory would allow. However, the L1 convergence order is about 4.5 at the last mesh-halving. Also note that the pressure accuracy is disappointing, as would be expected from the penalty method. Table 2 indicates that reasonable accuracy is retained even for moderately high Reynolds numbers and a small number of variables.

The second approach involved a modified penalty method, which holds the promise of yielding exact satisfaction of the incompressibility constraint as well as more accurate pressure approximations. Implementing the modified penalty method, together with Newton's method for treating the nonlinear term, required a substantial reworking of the TWODEPEP code. Convergence of the latter approach had been cast in doubt in a remark in the book AUGMENTED LAGRANGIAN METHODS by Fortin and Glowinski, and our investigations, as indicated in Table 1, tended to support this observation. Thus further investigation is needed to determine a method to link the modified penalty approach with efficient techniques for resolving the nonlinear terms.

The implementation of the streamline-diffusion, finite-element method for inviscid flows required starting a code from scratch. The first step was to develop a solver for a linear, hyperbolic equation using the streamline-diffusion, finite-element method. This is later to be coupled with an elliptic solver (e.g., Randy Bank's multi-grid code) to implement a full nonlinear Euler equation solver. However, the critical new component is the streamline-diffusion technique for temporal integration of an advection equation, so this was addressed first. Numerical tests were performed on this technique with the code. It appears that the code is very effective in limiting cross-wind diffusion for discontinuous solutions (see Figure 4), and thus shows promise as a component in a high-Reynolds-number code. Tests were also performed on problems with smooth data, cf. Figure 5. Convergence test for this problem indicate second-order convergence, and the L-2 error for the time t = 0.6 shown in Figure 5 was 0.012 . What remains to be done is the linking of the elliptic solver as described previously together with development of efficient techniques for solving some implicit equations that are inherent to the method at each time step.

TABLE 1

Convergence tests of the quartic finite element method on Jeffery-Hamel flow as shown in Figure 3. All runs with the same Reynolds number: Rave = 6.85, Rmax = 9.89.

# triangles used	velocity L1 error (relative)	velocity L2 error (relative)	velocity Max error (relative)	pressure Max error (relative)	See Key below
7	0.762E-2	0.131E-1	0.273E-1	n/a	A
7	0.871E-2	0.143E-1	0.288E-1	n/a	B
7	0.871E-2	0.143E-1	0.288E-1	0.200E0	C
28	0.676E-3	0.158E-2	0.453E-2	0.108E0	A
28	0.769E-3	0.177E-2	0.508E-2	0.122E0	C
112	0.342E-4	0.117E-3	0.565E-3	0.720E-1	A

Key to Methods Used

A straight penalty, epsilon = 1E-6, 3 stage continuation, 3 extra iterations with full nonlinearity

B straight penalty, epsilon = 1E-6, 3 stage continuation, iterated with full nonlinearity until "convergence"

C pressure only relaxed (by 1.2**-n at the n-th iteration) penalty iteration, epsilon = 1E-6, 3 stage continuation, iterated with full nonlinearity until "convergence"

TABLE 2

Errors for the quartic finite element method calculating Jeffery-Hamel flow for various Reynolds numbers, using only seven triangles, as shown in Figure 3a.

Reynolds number (Rave)	6.9	18	65	222
Relative L2 velocity error	0.13E-1	0.27E-1	0.69E-1	0.15E0

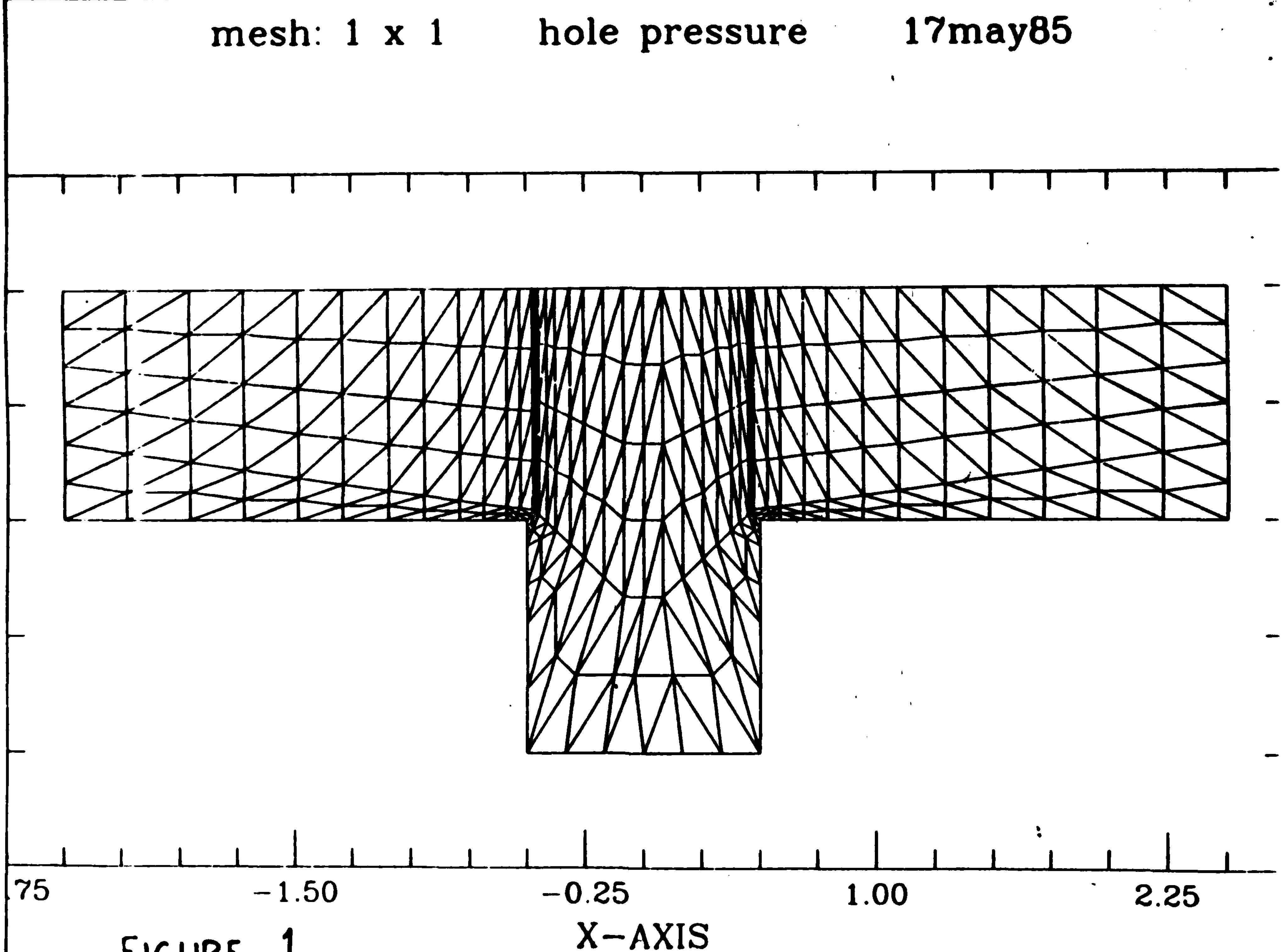


FIGURE 1.

depths 1,3,2&1.5 KMESH 1(x2,3extrap) 24sep85

○ □ △ ☆

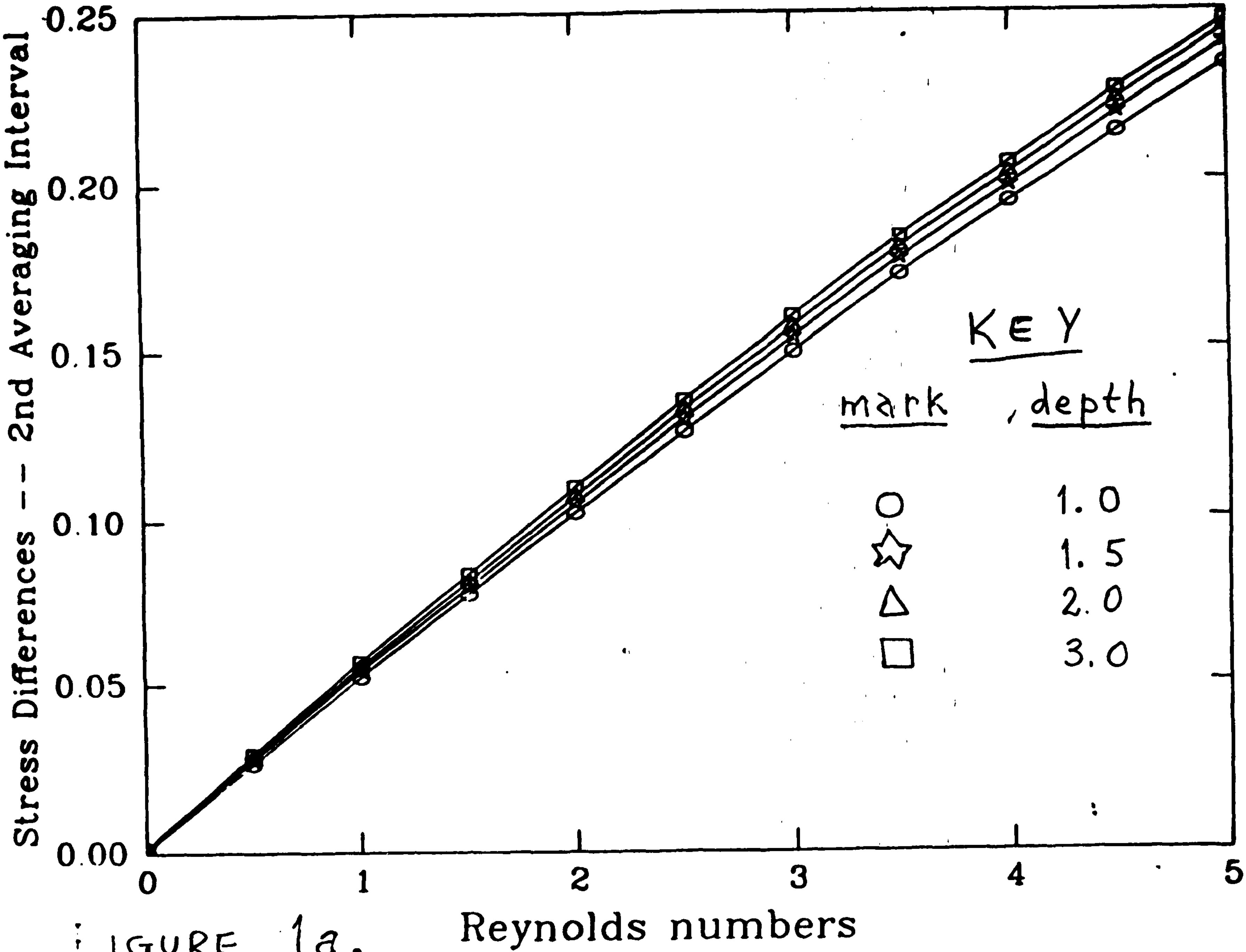


FIGURE 1a.

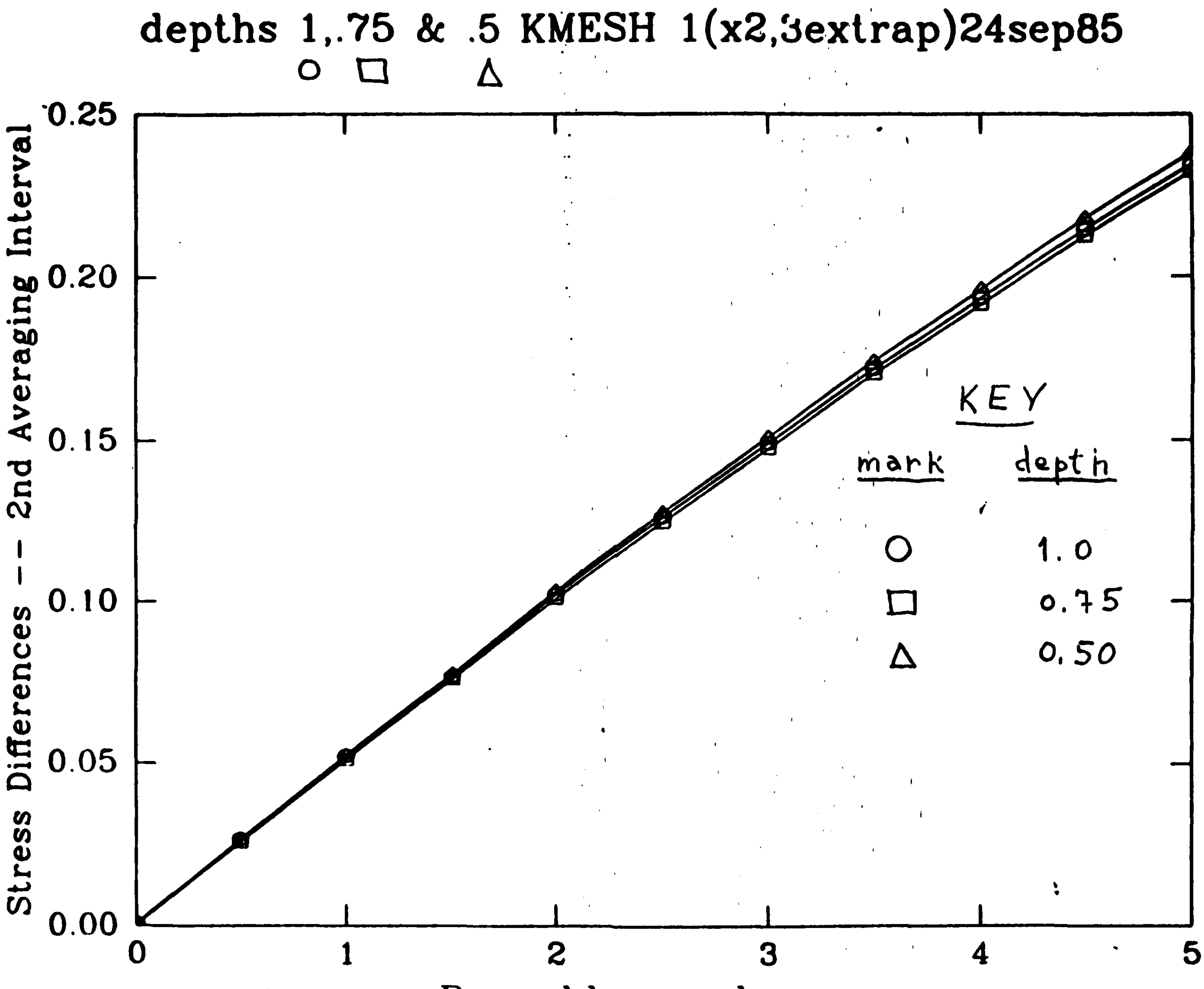
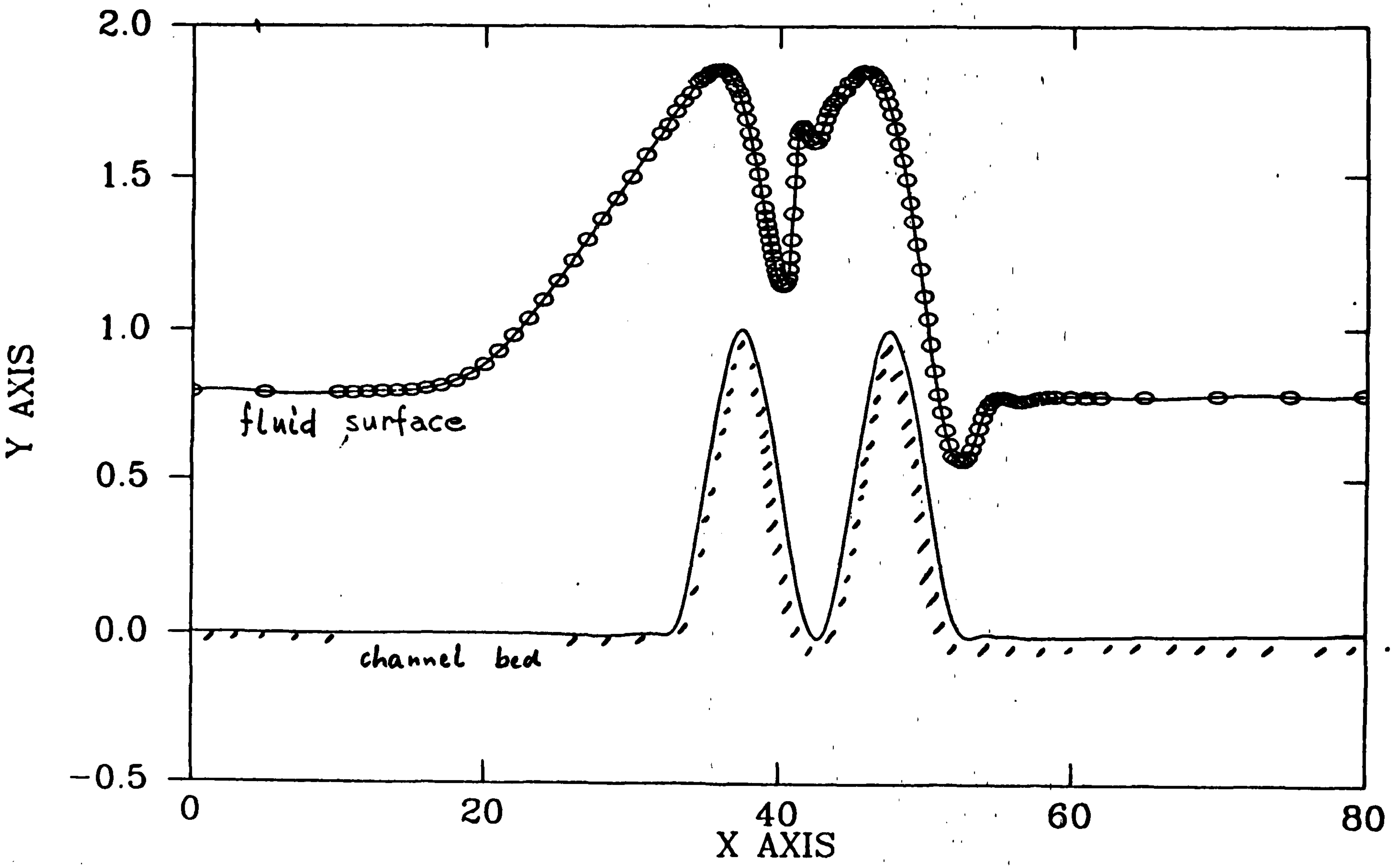

depths 1,.75 & .5 KMESH 1(x2,3extrap)24sep85
○ □ △
Stress Differences -- 2nd Averaging Interval
0.25
0.20
0.15
0.10
0.05
0.00
0
1
2
3
4
5
Reynolds numbers
KEY
mark depth
○ 1.0
□ 0.75
△ 0.50

FIGURE 1b.

EXPERIMENT NO. 6.

R = 20·5

channel slope = 0·0735 rad



DEPTH: 0.794 cm, VISCOSITY: 0.766 St.

FIGURE 2.

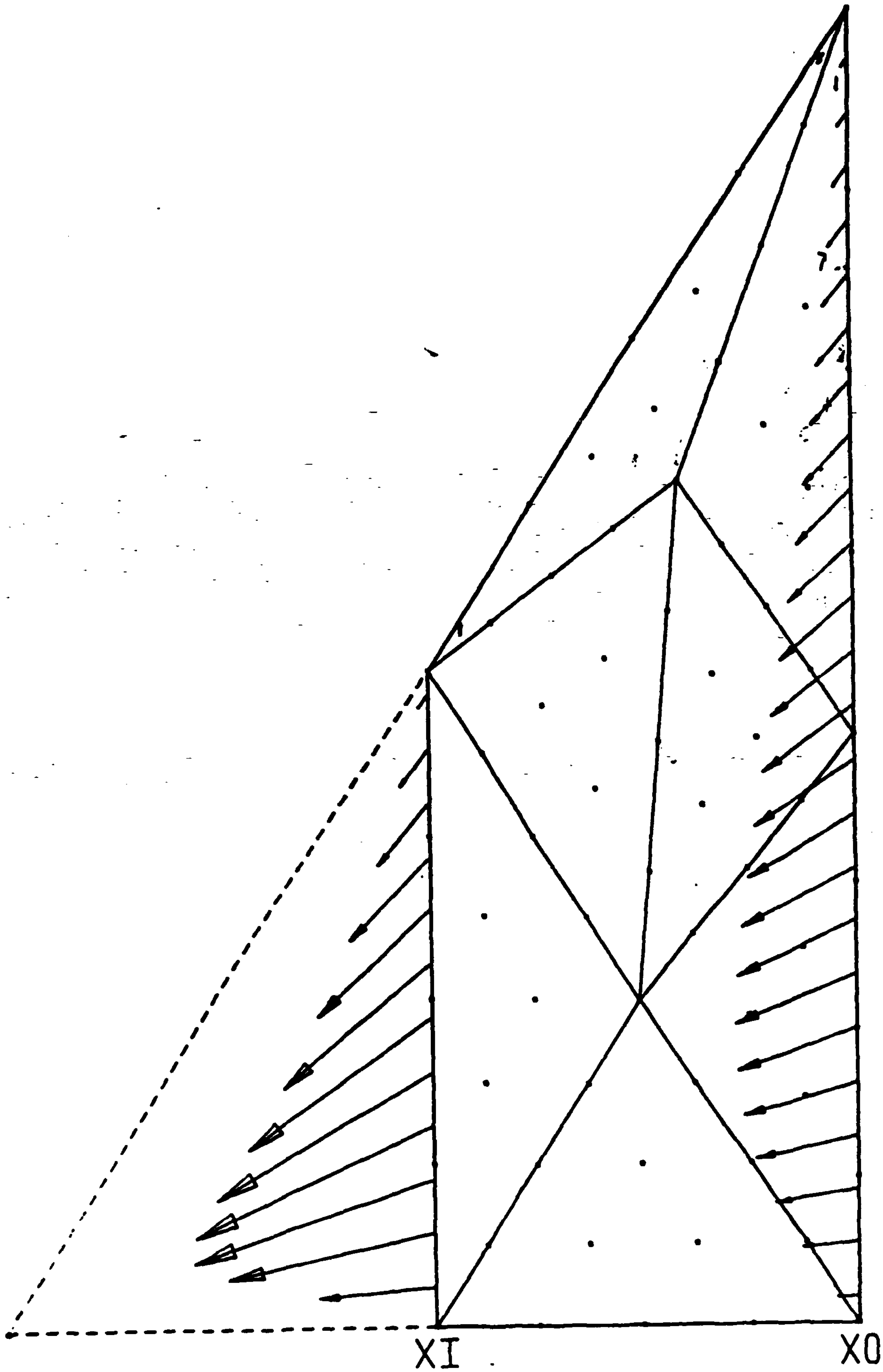


FIGURE 3a.

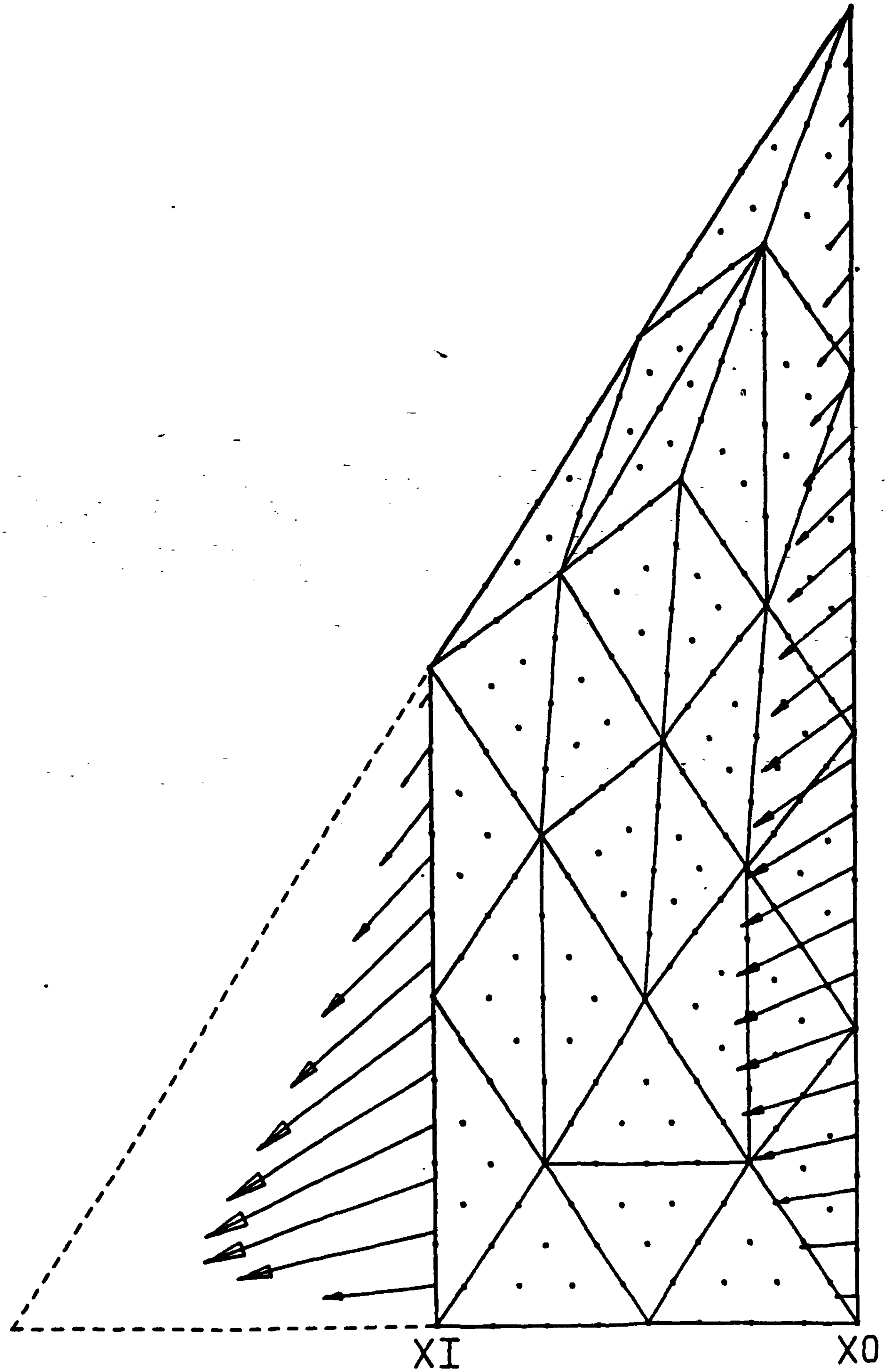
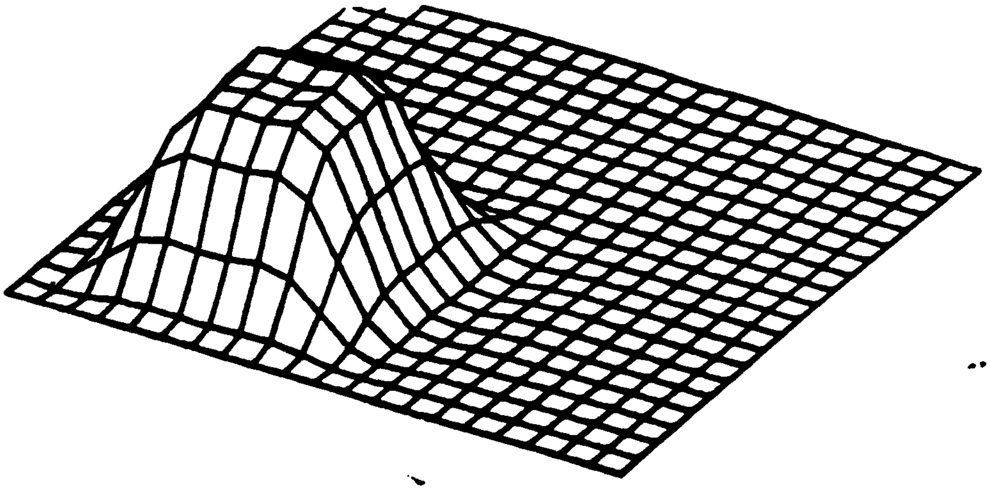
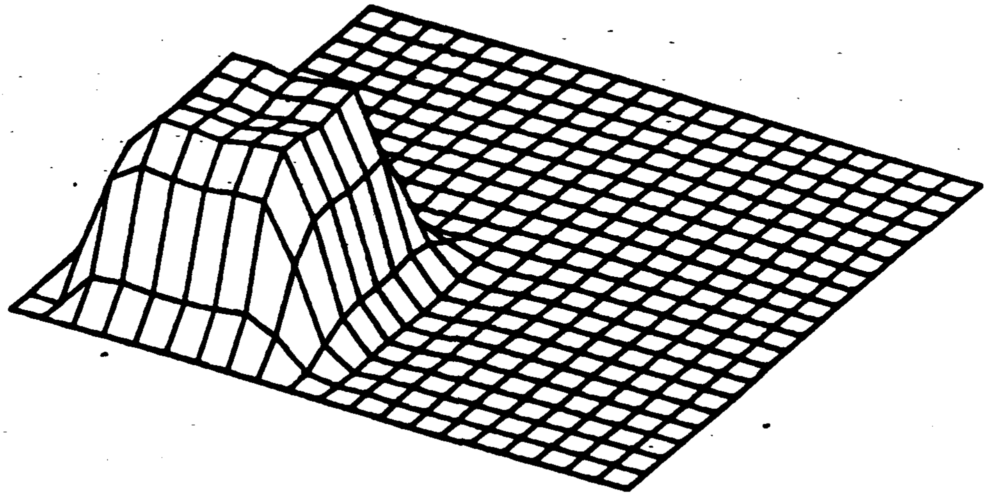


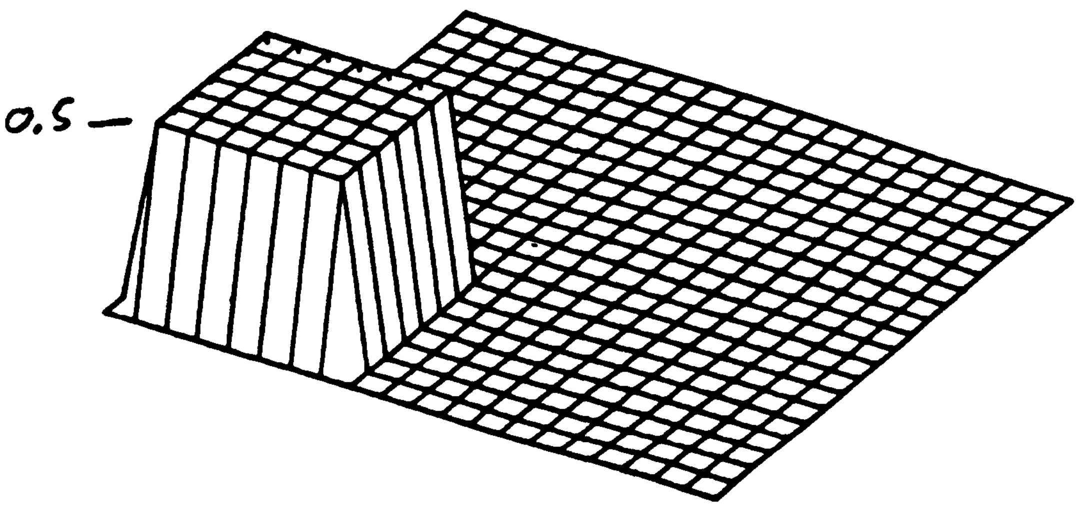
FIGURE 3b.



TIME = 0.100 SLAB = 2

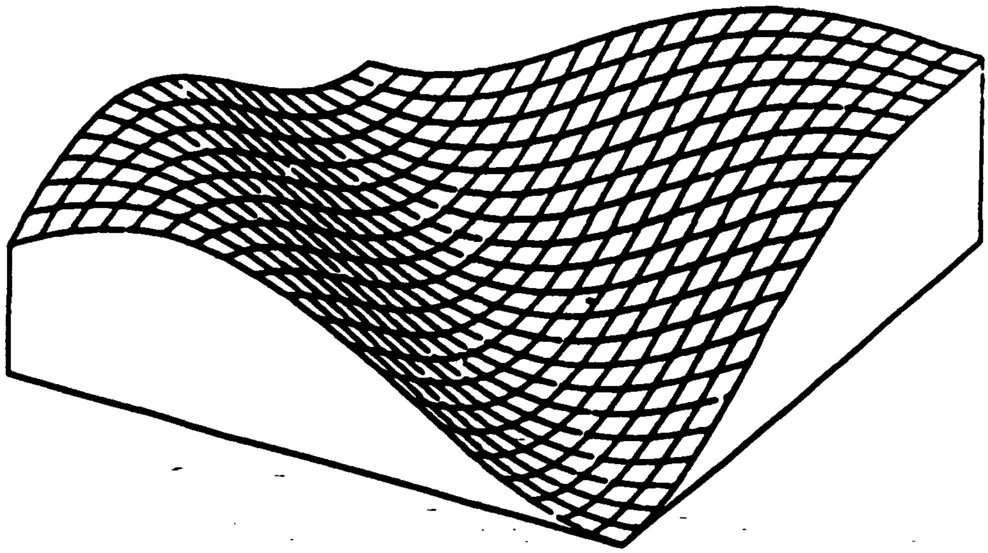


TIME = 0.050 SLAB = 1

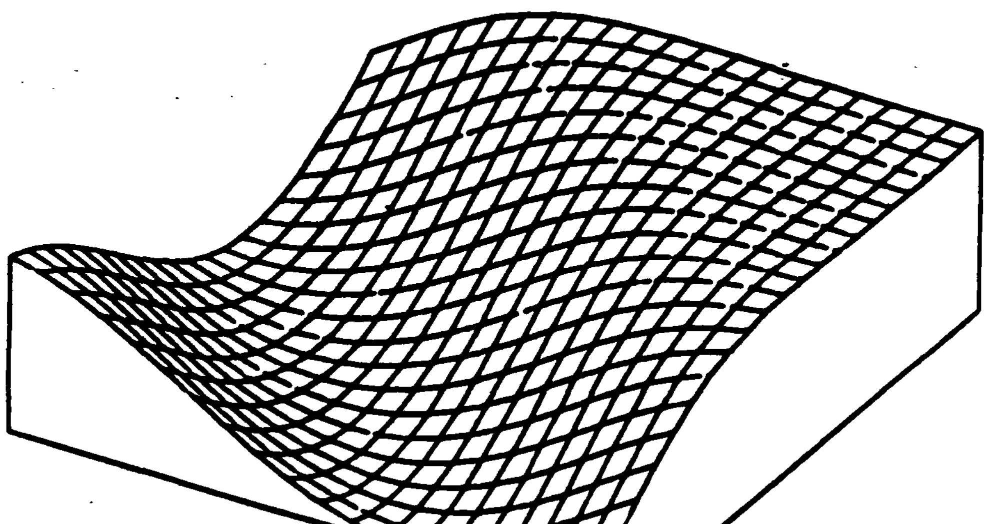


TIME = 0.0 SLAB = 0

FIGURE 4



TIME = 0.600 SLAB = 12



TIME = 0.400 SLAB = 8

FIGURE 5

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